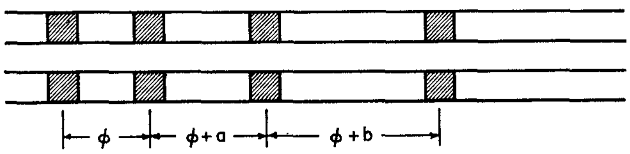
Discussion on Optimum Bead Spacing*

Mr. Dettinger's article[1] on the optimum spacing of bead supports in coaxial transmission lines is too sanguine as to the performance which will result. He states that an array of M beads arranged according to his theory will result in a total reflection coefficient of no more than $\sqrt{M}\Gamma_0$ where Γ_0 is the reflection of one bead. In his Fig. 5 he shows a chart of the reflection of four beads equally spaced and the reduced reflection which he hopes will result from application of his rule. The case where there are four beads will result for equal spacing in a maximum value of $4\Gamma_0$, and from his theory for progressive spacing it should be possible to arrange the beads so that the maximum possible reflection is only $2\Gamma_0$.

Assume that the reflection of each bead in Fig. 1 is very small so that the total reflection of the array of beads can be found by simple vector addition of the reflections of the individual beads. Also assume that the reflection from each bead is invariant with frequency and the differences in bead spacing a and b are constant with frequency. Then the reflection patterns of Figs. 2 and 5 of Mr. Dettinger's article become equivalent to plotting the pattern as a function of phase angle ϕ instead of frequency.



Fig

If now the value of the total reflection coefficient Γ_t as a function of ϕ is computed and this value is squared it will be found that the average value of this power reflected computed for one cycle is exactly $M\Gamma_0^2$. In this case $M=4$ but the statement is true for any M and is independent of a and b. Now let us imagine that we are able to space an assembly of beads so that the total reflection coefficient has a constant value over a cycle of ϕ. Of course this constant value of reflection coefficient would have to be $\sqrt{M}\Gamma_0$. But this is just the value of reflection coefficient which Mr. Dettinger claims is the *peak* for his spacing.

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Author's Comment[2]

I am sorry that the objectives of my paper were not clear to Mr. Reed. My goal was to reduce the envelope of reflection peaks in a limited frequency band for a long coaxial transmission line made up of many identical sections. The idea of the limited band was discussed in the text and figures, but was not explicitly restated in the summary or in the conclusions.

Mr. Reed has presented a concept regarding the average reflection from any number of beads arbitrarily dispersed along a line. This concept is only incidentally related to the subject of my paper, in that it considers neither the actual envelope of reflection peaks nor the possibility of reducing the envelope in a limited frequency band by the use of a controlled dispersal of beads in each section.

However, he has raised a philosophical issue. The question might be stated—Can the maximum value of a varying function be equal to its average value under any circumstances? I believe that the answer is yes in the special case where the maxima are measured in a frequency band which is narrow in comparison with the band over which the average is computed. This happens to be the very case we are considering. The bandwidth of reduced reflection envelope is necessarily limited by the requirement that the progressive increment remain close to its nominal design value, whereas the average reflection would be computed over a much wider band. Therefore, I believe that the two concepts are not inconsistent. Instead, Mr. Reed's work has yielded the interesting observation that the reflection may "pile up" in other parts of the frequency band.

It is perhaps necessary to point out that in his second sentence Mr. Reed has misstated my formula for the predicted reduction of reflection envelope. The relation he gives, $\sqrt{M}\Gamma_0$, is a simplified form obtained only for the case where M, the number of beads per section, is equal to N, the total number of beads in the array; in short, for one section. The general form would be $N\Gamma_0/\sqrt{M}$. This formula is not intended to apply to the case of one section and would require modification for that use.

As was indicated in my paper, a complete mathematical analysis of the progressive dispersal across a wide frequency band has not been achieved. It has been suggested that the solution may be arrived at through conventional antenna array theory. I hope to carry this through when time permits. In the meantime, I should welcome any contribution of another worker, such as Mr. Reed, to the mathematical solution of a relation which has been verified experimentally.

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* Received by the PGMTT, March, 1959.

[1] D. Dettinger, "The optimum spacing of bead supports in coaxial line at microwave frequencies," 1957 IRE CONVENTION RECORD, vol. 5, pt. I, pp. 250-253.

[2] Received by the PGMTT, April, 1959.

Concerning Riblet's Theorem*

I would like to add yet another comment[1,2] concerning Riblet's theorem.[3] Ozaki's impedance function

$$Z(p) = \frac{2p^2 + 2p + 4}{3p + 1} \quad (1)$$

which satisfies an equation of the form

$$m_1(p)m_2(p) - n_1(p)n_2(p) = C(1 - p^2)^n \quad (2)$$

(with $C=4$ and $n=1$), can be realized in many ways, of which only four are shown in Fig. 1. [Type (b) was the only one given by Ozaki and his numerical solution was incorrect.]

As is well known in lumped constant network theory, for a driving point impedance $Z(p)$ to be physically realizable, the degrees of its numerator and denominator polynomials can differ by at most unity. The same property carries over to resistor-transmission-line circuits by Richards' transformation.[4] It follows from Riblet's proof[3] of his theorem, that a rational impedance function $Z(p)$ which is positive real and satisfies an equation of the form (2) can be realized as a cascade of n equal-line sections terminated in a resistance and possibly one stub, as in Ozaki's example. As with the three circuits (a), (b) and (c) of Fig. 1, it can be seen that in general, if one stub appears at the termination, it can be moved in whole or in part, and placed at any one junction or shared out between several junctions anywhere along the cascade of lines. (Of course the characteristic impedances of the cascaded lines will change as the stubs are passed over them, but there will always be just n cascaded line sections, whereas the number of stubs may be changed.) The form $Z(p)$ determines the *minimum* number of stubs required, and Riblet's theorem may be generalized as follows:

> "The necessary and sufficient conditions that a rational function of p, written in its lowest form,
>
> $$Z(p) = \frac{m_1(p) + n_1(p)}{m_2(p) + n_2(p)}$$
>
> with m_1 and m_2 even, and n_1 and n_2 odd polynomials in p, be the input impedance of a cascade of n transmission line sections of equal length θ, terminated in a resistance with at most one study of length θ, are

1) $Z(p)$ must be a positive real function of p;
2) $m_1m_2 - n_1n_2 = C(1 - p^2)^n$.

> The numerical difference between the degrees of (m_1+n_1) and (m_2+n_2) is equal to the minimum number of stubs involved."

* Received by the PGMTT, May 22, 1959.

[1] H. Ozaki, "On Riblet's theorem," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 331-332; July, 1958.

[2] H. J. Riblet, "Comments on Ozaki's comments," IRE TRANS. ON MIROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 297-298; April, 1959.

[3] H. J. Riblet, "General synthesis of quarter-wave impedance transformers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-5, pp. 36-43; January, 1957.

[4] P. I. Richards, "Resistor-transmission-line circuits," PROC. IRE, vol. 36, pp. 217-220; February, 1948.